THE FOUNDATIONS OF MATHEMATICS AND THE MATHEMATICS CURRICULUM

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In teaching the foundations of mathematics within the framework of a Christian college, and particularly that of a Christian liberal arts college, there are two groups of students which must be served. The first consists of the non-mathematics majors--those non-scientifically oriented "general anything" students who, as a catalog might put it, are to receive "an introduction to and an appreciation of the history, foundations, culture and applications of mathematics." The second group consists of the mathematics majors, and the few science majors who have not been frightened away by the calculus. The gulf between these two groups is sufficiently large, I believe, to indicate the use of two different strategies.

It is not difficult to see the incongruity in the hypothetical catalog description. It is impossible to achieve this desired result in one or two freshman mathematics courses. But because of outside parameters, such as the 130 hours required for graduation, it is also impossible to reasonably expect more than a one- or two-semester exposure to mathematics for most students. The problem is compounded still further by the demands of students, and faculty, for relevancy (and what is less relevant than the foundations of mathematics?). And if that were not enough, more and more students are entering college without the basic arithmetic skills needed to function efficiently and effectively in today's world.

How do we teach foundations under these constraints? Do we want to teach foundations to these students? If we answer the second question negatively, there is no need to consider the first. But I am sure most of us can agree with the following statement taken from Christian Liberal Arts Education, Report of the Calvin College Curriculum Study Committee, 1970:

> "Mathematics today is an important part of the intellectual scene, both in its own right and in its use as "the language of science". It has also been important in many stages of Western History--classical Greece, the late Renaissance, the Enlightenment. Its methods and results are interwoven in the intellectual and technological history of the West. It displays a rigor of procedure not to be found in any other discipline. For these reasons, we recommend the continuation of the present requirement."

The problem emerges. We want students to take a mathematics course, to be exposed to mathematics. But there is no concensus as to what should be included in such an experience. Because of this, I feel that many students are getting just that--an experience. I would like to outline three possible solutions to this problem. I am familiar with all three because we have used all three of them at King's. The reason for the changes was (and is) partial success. I do not mean total dissatisfaction but a feeling that, given the "parameters", we could do something better.

When I started at King's, part of the core curricular requirement was a mathematics course entitled, "Nature of Mathematics". The syllabus included logic, set theory, number systems, mathematical method, axiomatic systems, and anything else the instructor could get in. It was offered in small sections, four each semester, with the only differentiation between sections being the instructor. The one exception was a section, offered in the fall, which was designated the majors' section. In this section more depth of material was presented. But even then credit was not given toward the mathematics major.

One of the advantages of this approach is the fact that you know all students are being exposed to the "proper content". (Whether or not you agree with our content is not important. Replace it with your content.) By offering sections with different instructors, the students were able to match their learning styles with the appropriate teaching style. Another advantage to this approach was that the other disciplines and instructors could rely on the fact that the students had been exposed to certain topics, such as logic, which would be useful in composition work.

An obvious administrative disadvantage is the manpower necessary to teach all those sections. To handle eight sections per year you need the equivalent of one full-time instructor. Another disadvantage, which is closely related to this, is the pressure of other introductory level mathematics courses. We found we had to offer a pre-calculus, integrated trigonometry and algebra course to prepare some of our students for the calculus. There was pressure to offer a discrete or finite mathematics course for the business majors. There was pressure to offer more in the way of content courses for elementary education majors. How can a small department offer all these options in addition to the general course? How can the students take another three or six hours of course work? In addition to these pressures, departments at King's were asked to reduce their faculty by the equivalent of one-half person. In light of these parameters, we felt we had to make a change.

The main feature of the alternative that was decided upon is that under the umbrella of one course title, "Nature of Mathematics", with the exception of a small core, content differs from section to section. We chose as our core: logic, set theory, mathematical method and axiomatic systems. We set up four distinct sections which we labeled Majors (M), Humanities and Education (A), Natural Science (N), Social Sciences (S).

In the Majors section, we attempted to go into much more depth in the core material plus adding material on number systems, abstract systems and history. We assumed our majors were prepared for the calculus, which wasn't always a good assumption. In the Natural Science section, in addition to the core material, we attempted to do the elementary function theory. With the core material, which we considered as essential, it was impossible to cover everything that is usually done in an elementary function course. We found we could cover polynomials, rational functions, logarithms, and exponential functions. The trigonometric functions were introduced, but usually only from one perspective--either as circular functions or triangular functions. Analytic geometry and complex numbers became casualties.

In the Social Science section, in addition to the core, we covered some of the usual finite mathematics topics: elementary probability and statistics, linear algebra and linear programming. We also tried to emphasize the logic and set theory more. In the Humanities and Education section, we covered the core material plus material on number systems, history and geometry. In reality, this section was exactly the same course as we had offered under the earlier plan.

The obvious advantage to this program is the ability to offer each student core curricular credit while at the same time offering something relevant, or at least closer to relevancy, but at the same time to expose him to methodology, history and some philosophical implications of mathematics. It offered the administrative advantage of two fewer sections overall per semester, and hence brought our department in line with the cuts that had been made. A disadvantage which some will be quick to point out is the sacrifice of some of the standard content of the standard introductory level course.

Within two years, the outside parameters at our institution changed again with a major revision of the college-wide core curriculum. These changes were made so that we might better achieve our Goals for the Student which include the following:

> "The student should have a reasonable grasp of the major concepts, principles and methods of inquiry in the principle areas within the social sciences, the natural sciences and the humanities. He should be able to see the place of each area in the structure of knowledge as a whole and in the Christian world view, to the extent that he can create meaningful and original relationships among ideas presented in the various areas and can see the relevance of Christianity to each area".

The changes which affected the mathematics department were the dropping of the requirement of a mathematics course and the institution of a new interdisciplinary course entitled, "Fundamental Issues in the Natural Sciences and Mathematics". The new course, offered at the junior-senior level, is wholistic in nature and interdisciplinary and integrative in approach. It deals with the issues of methodology and presuppositions and questions such as, "What is the nature of truth?" Within the framework of questions and problems from the natural sciences, the questions of mathematical truth, <u>a priori</u> or <u>a posteriori</u>, probability and nature of data arise naturally. This approach offers the advantage of placing these questions of the foundations of mathematics within the larger framework of the foundations of science and mathematics and a Christian worldview.

Another advantage is the reduced teaching load in terms of the course, "Nature of Mathematics". We are moving in the direction of "more relevant" service courses, which we hope will attract students. One obvious disadvantage is the possibility of a student not being attracted to any mathematics course. In which case, he can complete a program without a "formal" mathematics course.

Which of these three approaches is best? The first two had some shortcomings. The third has not had enough time yet to prove, or disprove, itself. Other institutions have used these and have been happy with them. For example, Oral Roberts University and Professor Verbal Snook were recently highlighted in Change's Report on Teaching: 3 with a program which had some similarities to the first outlined here. It appears that perhaps there is no ONE answer for all institutions, or even for a given institution over a period of time. But, given the institution and its parameters, there should be a program.

Now that we have settled the matter for the general student, what do we do with the mathematics majors? I am sure that most of us come from institutions and departments which have goals, written or implicit, similar to the following statement from our Goals for the Student:

> "The student should master the principles, techniques, and methods necessary to begin independent work and to make independent judgments in his major field of study. He should have a reasonable understanding of the extent of his major subject, its history, its relation to the other fields, and its place in contemporary culture."

Doesn't that statement commit us to help our students struggle with the foundations of mathematics? Consider another of King's stated goals:

> "The student should develop the ability to think logically, critically, and creatively. This ability entails attacking a new problem, translating it into workable terms, identifying central issues, recognizing underlying assumptions, evaluating evidence, drawing warranted conclusions, and proposing suitable solutions."

To achieve these goals, one cannot rely on one course. The whole curriculum, and each course individually within that curriculum, must be planned with them in mind. One senior seminar is not enough. Without a foundation from all courses, one experience is insufficient. I would like to outline our curriculum and indicate how we feel each point contributes to the ultimate goal. Our majors are required to take 32 hours of mathematics, 12 hours of which is our calculus sequence. This is the foundation of our analysis courses, and is a prerequisite for over half of our majors' courses. In it we hope to acquaint and introduce the student to the process of analytic thinking. This introduction need not be a "formal" one. It can be best accomplished by the general tone of the course, a few well-chosen comments, examples, and problems throughout the course.

Our next majors' requirement is a three-hour axiomatic course chosen from Algebraic Structures, Geometry, Advanced Calculus, or Topology. The emphasis of these courses is on the structure of the subject matter, and in some sense on structure itself. This provides a reference point for the student in later discussions on the nature and foundations of mathematics.

Our majors have 12 hours of mathematics electives which they may choose from Linear Algebra, Probability, Statistics, Computer Science, Introduction to Applied Mathematics, Numerical Analysis, or the other axiomatic courses. The electives should also have input into the foundations question. For example, the concept of modeling and the modeling process are emphasized in Introduction to Applied Mathematics. The computational, manipulative "what works" aspects are emphasized in Numerical Analysis. These are explicit goals of these two particular courses, stated in writing in the course objectives in the syllabi. But I don't think they have to be "preached down the students' throats". Again, it can be by the general tone of the course, the choice of textbook, the choice of supplementary material, whether lectures, assignments, or readings. The student should be able to sense it, if not at the time of the experience, at least upon guided reflection.

You will recall that I said our program consisted of 32 hours. And if you have been counting, you would know I have accounted for 27 hours. Two of those remaining hours are the Senior Seminar, which is run on the outline developed by Harold Heie and described in his paper. What are the other three? The three hours represent a requirement which I feel is somewhat unique as a requirement. We require a junior-senior level course which is entitled, "History and Foundations of Mathematics".

Obviously a one-semester course in the history and foundations of mathematics is a tall order. This course is meant to be an introduction to the historical and foundational framework of mathematics. The development of mathematics did not occur in a vacuum. Technological, political, and sociological developments influenced mathematical developments, and vice versa. An appreciation of mathematics, or a branch of mathematics, is not complete or realized without some grasp of these interrelationships.

This course is not just a "history" course though. It is a "mathematics" course and "significant mathematics" is done throughout the course through the use of appropriate problems and lecture demonstrations. For example, in the discussion of Hamilton, we went through a development of quarterions in class.

For our purposes, I think we have found an excellent text. It is <u>An Introduction to the History of Mathematics</u>, 4th ed., by Howard Eves. I have had to supplement certain "technical aspects." But each chapter concluded with problems which helped the students crystalize the historical concepts and see the difficulties and subtleties that are involved. How many of you have used Fermat's method, or Barrow's method, or Newton's method of fluxions to calculate the slope of the tangent to a curve? That is one of the exercises at the end of the chapter on the development of the calculus.

In addition to problems which were assigned, another major requirement of the course was a paper on a particular mathematician. This paper was to give a biographical sketch, a description of the historical setting of the individual and an analysis of some of the major works of the individual. It was to indicate the influences which affected the individual, and also the effect the individual had on mathematics and society.

The course, although still in the developing stages, has been extremely well-received by our students. One student, who had been wavering between further study in mathematics or chemistry, decided on mathematics. He said that he could identify with some of the thought processes of some of the historical mathematicians. He had not been able to do that in chemistry.

That is our program. What I hope I've indicated to you is that it need not be your program. But you should have a program, a well-thought out program, a working program.

The King's College
Instructor: Dr. Bayard Baylis

Course: History & Foundations of Mathematics (MA 452)
Spring 1977

TEXT: Howard Eves, An Introduction to the History of Mathematics, 4th ed., Holt, Rinehart & Winston (1976)

COURSE OBJECTIVES: Since mathematics is centuries old, a one-semester course in its history and foundations cannot even begin to scratch the surface. This course is meant to be an introduction to the integration of mathematics into its historical framework. The development of mathematics did not occur in a vacuum. Technological, political, and sociological developments influenced mathematical developments, and vice versa. An appreciation of a branch of mathematics is not complete without a grasp of these interrelationships.

Also, since this is a "mathematics" course, a considerable amount of mathematics will be infused throughout the course through problems. By working out problems, the important historical concepts will become more crystalized, the difficulties and subtleties appreciated and understood. They can provide much material for future teachers.

CLASS PROCEDURES AND GRADING CRITERIA: The classes will be lecture-demonstrations, although it is hoped that much discussion will occur. There will be no in-class examinations. For each "period" covered, problems will be assigned, due one week after assignment. There will be a comprehensive final.

Each student is to select a mathematician and in a paper (minimum length 1500 words) describe the historical setting in which the mathematician lived and worked, give a brief biographical sketch, and an analysis of one or two of the major results. It should point out the influences which affected the mathematician, and also what effect the mathematician had on mathematics and society.

The weightings are as follows:

Assigned problems	30%
Paper	40%
Comprehensive Final	30%

COURSE OUTLINE:

I. Pre-Seventeenth Century
   A. Numerical Systems
   B. Babylonian
   C. Egyptian
   D. Greek
      1. Pythagorean
      2. Euclidean
      3. Post-Euclidean
   E. Far Eastern
   F. European

II. Modern
   A. Seventeenth Century European
   B. Analytic Geometry
   C. Calculus
   D. Eighteenth Century European
   E. Nineteenth Century European

REFERENCES:

1. Blunt, Jones & Bedient, The Historical Roots of Elementary Mathematics, Prentice Hall
2. Boyer, A History of Mathematics, Wiley
3. Eves & Newsome, The Foundations and Fundamental Concepts of Mathematics, Rev. Ed., Holt, Rinehart & Winston
4. Kline, Mathematics in Western Culture, Oxford
5. Kline, Mathematics - A Cultural Approach, Addison Wesley
6. Kramer, The Nature and Growth of Modern Mathematics, Fawcett (paper, 2 vols.), Hawthorne (cloth)
7. Kramer, The Main Stream of Mathematics, Oxford
8. Resnikoff & Wells, Mathematics in Civilization, Holt, Rinehart & Winston
9. Newman, The World of Mathematics, 4 vols., Simon & Schuster
10. Scott, A History of Mathematics, Barnes & Noble
11. Smith, Source Book in Mathematics, 2 vols., Dover
12. Struik, A Source Book in Mathematics, 1200-1800, Harvard
13. Turnbull, The Great Mathematician, Simon & Schuster
14. Weiner, I am a Mathematician; and Ex-prodigy, MIT Press
15. Wheeler, Josiah Willard Gibbs, Yale
16. Wilder, Introduction to Foundations of Mathematics, 2nd Ed., Wiley
17. Wilder, Evolution of Mathematical Concepts, 2nd Ed., Wiley

The King's College
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Course: Integrative Seminar (MA 462)
Spring 1977

COURSE OBJECTIVES: To initiate within the student, an attempt at personal integration of mathematics and science as a whole into the structure of knowledge as a whole, and particularly, into a Christian world-view. To deal with some of the significant questions dealing with the foundations of mathematics, science, knowledge and Christianity.

CLASS PROCEDURES: We will meet twice weekly for discussion centered on a pre-assigned reading. Questions that may serve as guides will be provided for each reading, although students are encouraged to develop their own discussion questions.

GRADING CRITERIA: The course grade will be determined by constructive participation in discussions (25%) and by a paper (75%), due Reading Day, in which the following points should be discussed:

1. Is mathematics "true"? What is the nature of "truth"?
2. Is mathematics an art or a science?
3. Is mathematics <u>a priori</u> (predictive) or <u>a posteriori</u> (descriptive)?
4. Is mathematics created or discovered?
5. Compare and contrast the methodology of a mathematician, a scientist, and a Christian apologist.
6. What is the place of mathematics in your view of total reality?
7. State your present plans for the future. Attempt to justify your plans in light of your present system of values.

REQUIRED READINGS:

1. G. H. Hardy, <u>A Mathematician's Apology</u>, Cambridge University Press (1973)
2. C. S. Lewis, <u>Miracles: A Preliminary Study</u>, Macmillan (1976)
3. Alfred Renyi, <u>Dialogues on Mathematics</u>, Holden-Day (1967)
4. William Schaaf, ed., <u>Our Mathematical Heritage</u>, rev. ed., Collier (1963)
5. G. Joseph Wimbish, <u>Readings for Mathematics: A Humanistic Approach</u>, Wadsworth (1972)

RECOMMENDED READINGS:

1. Barbour, <u>Issues in Science and Religion</u>, Prentice-Hall
2. Barker, <u>Philosophy of Mathematics</u>, Prentice-Hall
3. Benacerraf & Putnam, <u>Philosophy of Mathematics: Selected Readings</u>, Prentice-Hall
4. Beardsley, <u>Philosophic Thinking, An Introduction</u>, Harcourt, Brace & World
5. Bell, <u>Development of Mathematics</u>, 2nd ed., McGraw Hill
6. Boyer, <u>History of Mathematics</u>, Wiley
7. Bronowski, <u>Science and Human Values</u>, Harper & Row
8. Frankena, <u>Ethics</u>, Prentice-Hall
9. Hadamard, <u>Psychology of Invention in the Mathematical Field</u>, Dover
10. Hawkins, <u>The Language of Nature</u>, Freeman

11. Howkins, The Challenge of Religious Studies, IVP
12. Kline, ed., Mathematics in the Modern World: Readings from Scientific American, Freeman
13. Kramer, Modern Mathematics: Its Growth and Nature, Hawthorne
14. Luijpen, A First Introduction to Existential Phenomenology, Duguesne Press
15. Mackay, The Clockwork Image, IVP
16. Mendelbaum, Philosophical Problems, Macmillan
17. Newson, Mathematical Discourses: The Heart of Mathematical Sciences, Prentice-Hall
18. Pedoe, The Gentle Art of Mathematics, Macmillan
19. Russel, Introduction to Mathematical Philosophy, George Allen & Urwin, Ltd.
20. Saaty & Weyl, The Spirit and Uses of the Mathematical Sciences, McGraw Hill
21. Stabler, Introduction to Mathematical Thought, Addison-Wesley
22. Standen, Science is a Sacred Cow, Dutton
23. Sullivan, The Limitations of Science, New American Library
24. Weiner, God & Golem, Inc., MIT Press
25. Wilder, Introduction to the Foundations of Mathematics, Wiley
26. Wittgenstein, Remarks on the Foundations of Mathematics, Macmillan